Vibration suppression by designing and fine-tuning the Tuned Mass Damper (TMD)

***Abstract* - Vibration mitigation is a necessary step in designing a functional system having multiple masses residing on or within it. Often working conditions can be said to be dynamic and involves interaction of these several masses. The masses, often referred as 'Degrees of Freedom' in pure mechanical terms, when interact among themselves or external forcing stimuli, results in the phenomenon of *Resonance* where the system vibrates severely and might lead to rupture or failing of the structure. Hence, it is desired that such severe magnitudes of vibration be reduced wherever the phenomenon of resonance is observed. There are several methods to achieve this task, namely Vibration Isolation, and Vibration Suppression. The first method aims at completely isolating the desired component or limiting the path for transmission of vibration to that component. The second method, which is the aim of our study, deals with addition of damping which would dissipate the vibration energy by transforming it into heat. The method is even more enhanced when an appropriately tuned feedback loop is employed in the system.**

Introduction

To suppress the vibration amplitudes that the structure is observing, we must realize that there exists a phenomenon of Resonance, and that the resonance is depended upon the number of Degrees of freedom that the system comprises of.

Consider a Multi-degree of freedom mass-spring-damper system having multiple masses interacting or responding to the primary forces that are acting/applied on it. Let us assume that the forcing input is a harmonic sinusoidal input with a particular driving frequency. The MDOF system can have natural frequencies equal to number of degrees of freedom which can be found out from eigenvalue problem of its mass-normalized stiffness matrix. If the system finds its one of the natural frequencies to be equal to the input driving frequency, then there is resonance in the system observed at that i^{th} natural frequency. That is where the amplitude peaks indefinitely until the phase shift changes the interacting frequencies. To depict the picture clearly, let us evaluate a simple Multi-degree of freedom system and try different vibration absorption scheme with different cases of Passive and Active vibration absorption. We would try to observe a physical system which is untuned and is vibrating under the application of unity sinusoidal force at one of the masses. Then, the next set-up would be a Passive Vibration

Absorption system where we append additional suppression mass to the system with the aim of further reducing the vibration amplitudes. Lastly, we would tune the entire system using Feedback gain control loop making it the fully tuned Active Vibration absorption system.

Quarter Car model - A case study

A normal engineering practice is to design the parameters of a full-scale car by performing the analysis onto a small segment of the car, the quarter portion. We can relax some of the conditions of symmetry to make the calculations easy.

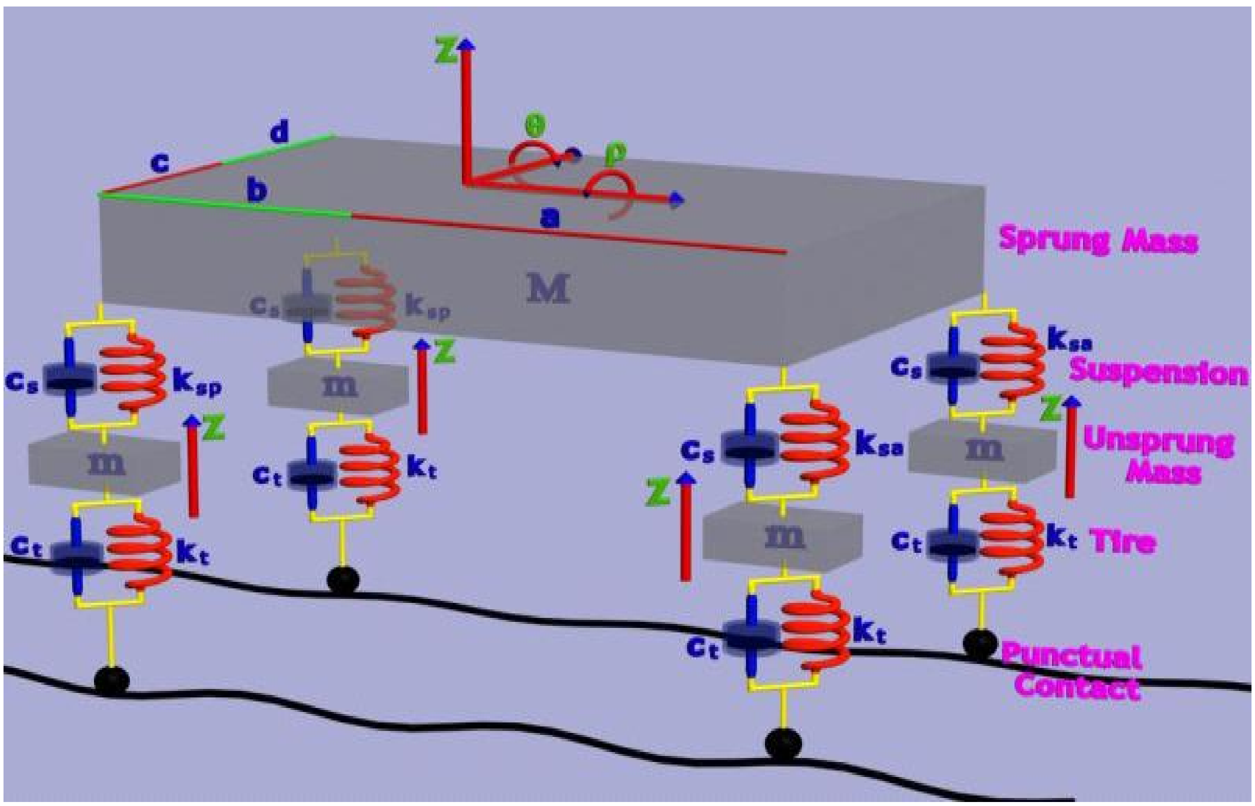

Figure 1: A full scale dynamic model of a car (Vincenzo Punzo)

The dynamic model of a full-scale car shown above includes important properties of tire dynamics as well. In order to design the unsprung portion of the car, it is best to consider the tire as having damping property along with the stiffness characteristics.

The scope of study in this review paper is focused on a performance car, specifically a racing car. The properties of a Race-tune cars are different from normal cars used in daily activities. Racing cars are supposed to not remain airborne for a longer time after hitting an irregularity on the road. This is achieved by having a spring-rate which is much lower than the stock specs. Spring which ensures regaining the contact to the ground more immediately. A lower spring rate at the unsprung level would mean that higher amount of vibrational energy will be transmitted across to the sprung mass. And, it is also not practical to let these vibrations transmit to the sprung mass. Hence, often the spring-rates at the sprung level are kept high or equal to the spring-rates of the tire.

Primary Structure Parameters

We are considering a quarter portion of the car system dynamics, which is essentially a 2 DOF system in its primary form. To perform analysis on the quarter section of the car, let us study how the natural frequencies of these 2 DOF system is posed over a range of natural frequencies. We would compute the natural frequency of the system at hand using the mass-normalized stiffness matrix of the system using eigenvalue problem. In the figure below;

m_{us} , m_s = Unsprung and sprung masses, respectively.
k_{us} , k_s = Spring rates at the unsprung and sprung level.
c_{us} , c_s = Damping coefficients at the unsprung and sprung levels.
x_{us} = Coordinate to track displacement of mass (m_{us})
x_s = Coordinate to track displacement of mass (m_s)
F_o = Force from irregular road profile on masses.

The ultimate source of vibration is the ground where the tire system maintains a continuous contact. *Sprung* masses include all that lies above the tire and the suspension system. *Unsprung* mass includes the tire and its fixtures.

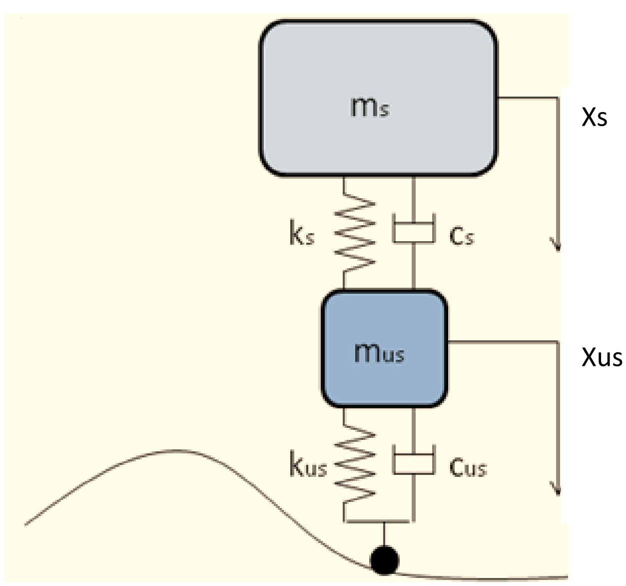

Figure 2.) Schematic diagram of a Quarter Car 2DOF system

The equation of motion for the above 2 DOF system is given in the matrix form as;

$$\mathrm{M}\ddot{\mathbf{x}} + \mathrm{C}\dot{\mathbf{x}} + \mathrm{K}\mathbf{x} = 0$$

In more specific terms;

$$\begin{bmatrix} \mathrm{Mus} & 0 \\ 0 & \mathrm{Ms} \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} \mathrm{Cus} + \mathrm{Cs} & -\mathrm{Cs} \\ -\mathrm{Cs} & \mathrm{Cs} \end{bmatrix} \dot{\mathbf{x}} + \begin{bmatrix} kus + ks & -ks \\ -ks & ks \end{bmatrix} \mathbf{x} = \begin{matrix} \mathbf{Fo\ sin\ \omega t} \\ \mathbf{0} \end{matrix}$$

Where ***x*** **=** $[x_1\ x_2]^T$ is the displacement vector

The following specifications are chosen for the primary structure analysis which is a quarte car model without any type of vibration absorption arrangement;

Parameters	Values
Sprung mass, m_s	120 kg
Unsprung mass, m_{us}	30 kg
Spring-rate suspension, k_s	9000 N/m
Spring-rate tyre, k_{us}	9000 N/m
Damping coefficient, *c*	[αM + βK] Proportional damping of 1%
Damping ratio, *ξ*	1 %
Force, *F(t)*	1 N upwards at unsprung mass

As the original system has 2 Degrees of Freedom, the natural frequencies obtained from mass normalizes stiffness matrix are;

$$\boldsymbol{\omega 1 = 5.9300 \quad rad/sec}$$

$$\boldsymbol{\omega 2 = 25.2950 \quad rad/sec}$$

Justification for the chosen parameters

Since we are focusing our analysis on a performance race car, we must realize how they are different from a daily-use car, in the sense that certain performance parameters are tweaked. For example, the cabin weight of a normal use car would lie in the range of 800 kgs and the suspension stiffness is of the order of 1/100th in comparison to the stiffness of the tyre. But, the case of Performance race cars is different. They are required to make sharp turns and must run flatter to the ground without much pitching or rolling. Hence, the spring-rate at the tyres are kept low to ensure that am immediate contact to the ground is regained after hitting an irregularity. On the contrary, the spring rate at the suspension is kept appropriately high in comparison to tyre stiffness. The reason being the car should run almost parallel to the road without undergoing rolling and pitching motions while negotiating the turn or immediate changes in the direction. Also, weight of the whole car is a major limitation to its intended performance. Therefore, the sprung weights are found out of the order of 300 – 700 kgs. And normally the unsprung mass is the mass that the tyre assembly has. And we are assuming that the system has light damping, hence the value of 'ξ', damping ration is taken as 0.01 for the primary structure.

To gain insight into the behavior of the system, we must perform the modal analysis. Modal analysis is the important design technique that characterizes the solution of the dynamic properties in the frequency domain. Often the mechanical systems have damping, and it becomes numerically more complex to compute the eigenvalues of the system. To model such large-order system, we utilize State Variable representation to gain the perspective of the system behavior from frequency response plots.

Frequency response plots, also known as Bode Plots shows peaks at each mode that the system has because of the number of degrees of freedom. These peaks are basically asymptotes occurring at these modes of vibration. Along these asymptotes, we will be able to measure the magnitude of the vibration that each mass is undergoing when a certain natural frequency is hit. To verify that all the masses will vibrate at that particular natural frequency, we should observe bumps coming from each mass on the plot on that frequency value.

Based on how symmetric the values of each mass are, usually the 1st mode is the one that vibrates with the highest magnitude. For the designers of a structure, it is an indication that that particular mode must be suppressed with addition of an absorber mass. What the absorber mass would do is that it would absorb a significant amount of vibrational energy and will undergo oscillations but within certain practical limits.

Before we tune the absorption system, let us look at the bode plots of our original system. Ours being a 2 DOF system in its original form will show two sharp spikes corresponding to two natural frequencies of the system. The frequency response plot (bode plot) is in meters vs rad/sec.

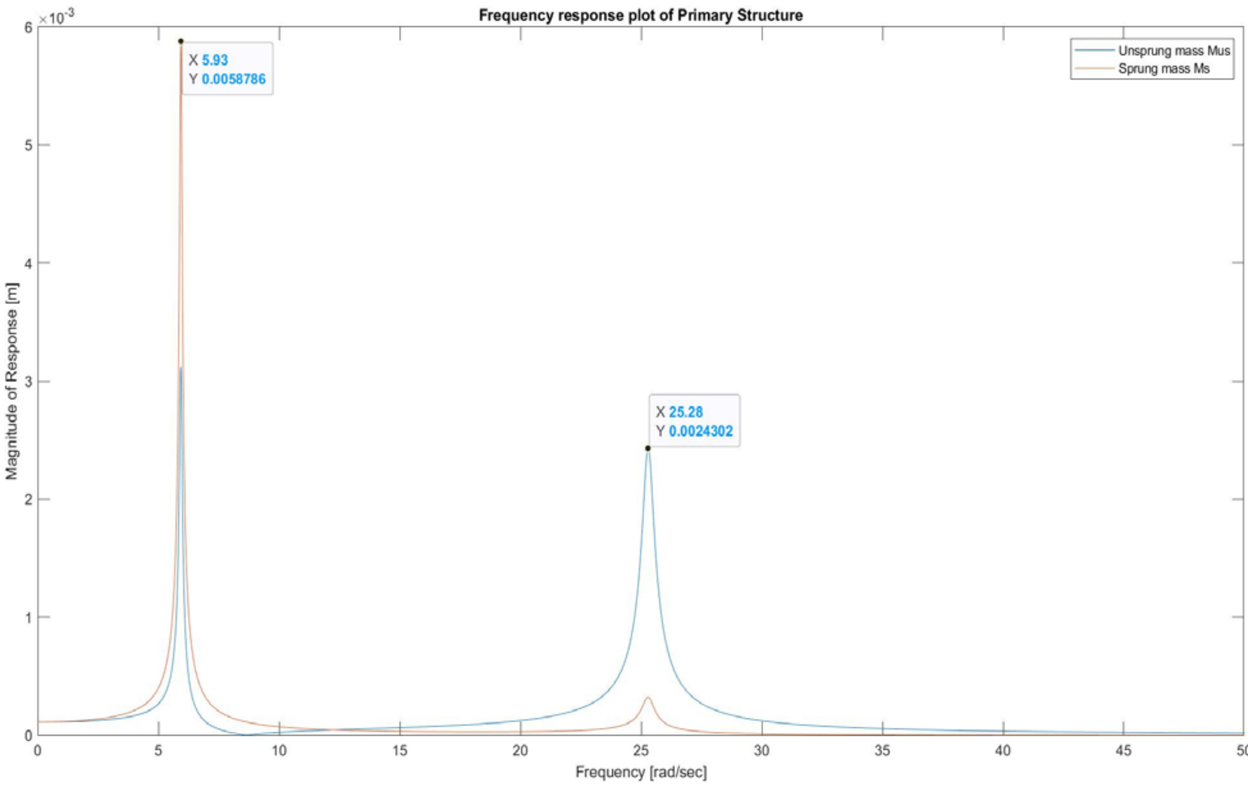

Figure 3.) Frequency response plot of the primary structure

The first resonance spike occurs at ω1 with sprung mass having higher amplitude of vibration than unsprung mass of order of 5.8786 mm. Ideally, we would like to suppress this amplitude and make it approach to zero theoretically. The other spike occurs at ω2, where the unsprung mass bounces higher than the sprung mass.

By changing the damping factor 'ξ', damping coefficient 'c' and spring-rates 'k', one can observe several changes in the way the resonant spikes are located along the bandwidth of frequency. The damping factor of 1% throughout the system suggests that there is light damping in the structure. This can be understood intuitively by the fact that the frequency responses are sharp and the values are in significant number of millimeters. For damping factor of 12 %, the overall amplitudes of the vibration of masses goes down. The response plot is attached below. The damping value's choice is determined by how much damping is affordable vs how much damping is necessary. For cars and other automobiles, there are weight and space constraints and thus increasing the damping would take a toll on the curb weight of the structure. We are concerned with those regions where these modes of vibration occur, other regions where the vibration of masses are running out of phase are not of importance.

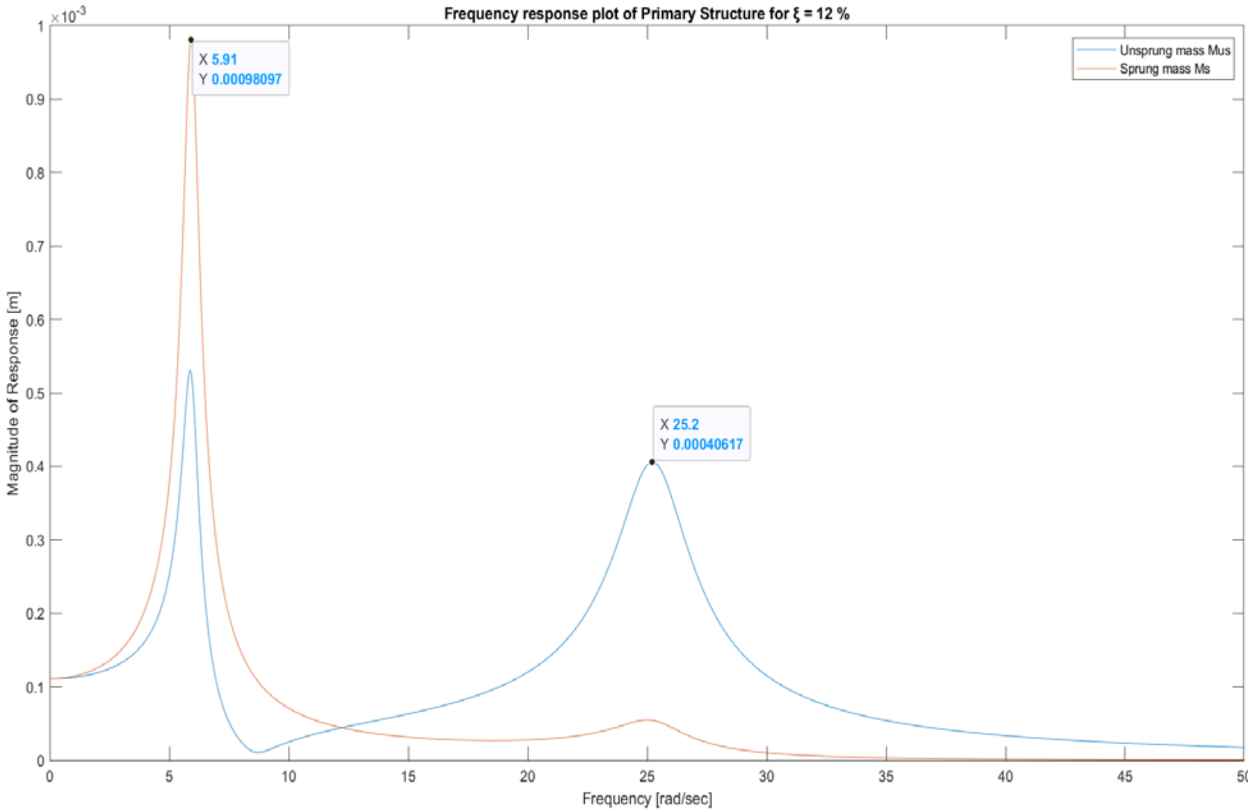

Figure 4.) Frequency response plot of the primary structure with 12 % damping

Design for Vibration Suppression

Passive Vibration Absorption System

Addition of extra mass is going to put up more weight in the structure. Hence, care is taken that the absorber's mass is not beyond 50 % of the primary structure's mass (case only for cars and Automobiles where passenger and other weights are accounted for) otherwise the effort will be redundant as the overall mass will increase by an order of half. Also, while tuning an absorber mass, one must be wary of the location where it is to be fixed.

Let the mass of the absorber added in our case be equal to 50 kgs. The tuning of passive system can be done in two ways. One case, where there is a fully tuned passive vibration absorption system, the other case which has no damping attached to the absorber mass.

A.) Fully-tuned Passive absorption case;

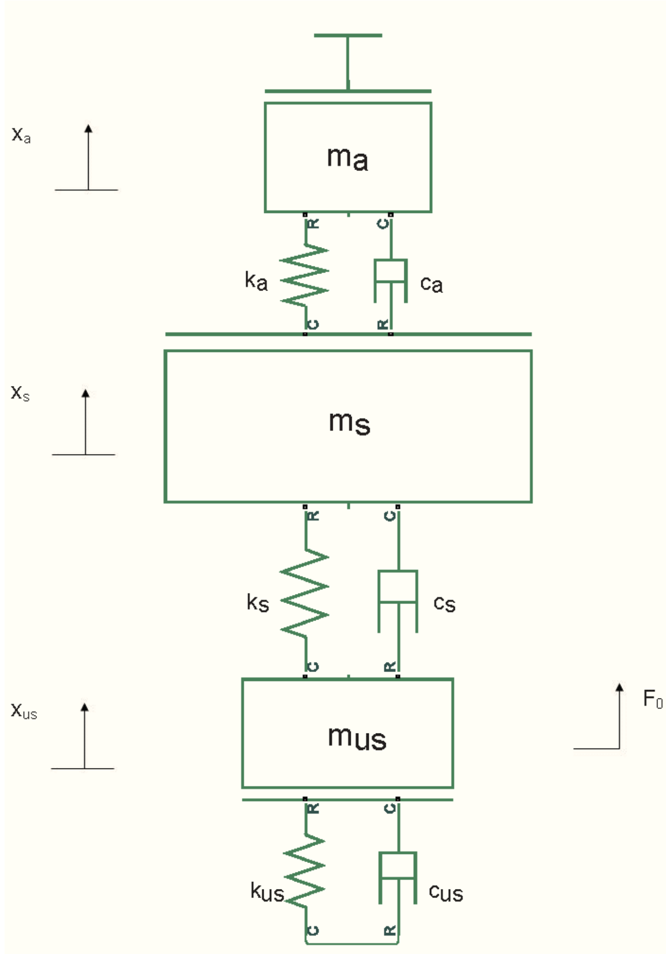

Figure 5.) Schematic diagram – Passive vibration absorption system (fully-tuned)

In this case, there is an additional mass-spring-damper on top of the Sprung mass. When fully tuned, the combined effect of both the damping and stiffness at the absorber level factors-in to suppress the 1st mode of vibration that occurs around the asymptote of $\omega 1 = 5.9300 \text{ rad/sec}$. But to target the suppression of the first mode is ensured by adopting appropriate value of absorber's mass, $\mathbf{m_a}$, and then approximating the spring-rate at the absorber mass level from it. The calculations are performed as;

$$\mathbf{k_a = m_a \omega_1^2}$$

Thus, the absorber mass (basically absorber mount) must be having the stiffness k_a. Since now we have added an extra mass, $\mathbf{m_a}$, the system is now a 3 DOF model. Appropriate changes in the Equation of motion of these 3 DOF system must be made. The 1 N force is still being applied by the road profile on the unsprung mass.

The following equation of motion is registered;

$$\begin{bmatrix} \mathrm{Mus} & 0 & 0 \\ 0 & \mathrm{Ms} & 0 \\ 0 & 0 & \mathrm{Ma} \end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix} \mathrm{Cus}+\mathrm{Cs} & -\mathrm{Cs} & 0 \\ -\mathrm{Cs} & \mathrm{Cs}+\mathrm{Ca} & -\mathrm{Ca} \\ 0 & -\mathrm{Ca} & \mathrm{Ca} \end{bmatrix}\dot{\mathbf{x}} + \begin{bmatrix} kus+ks & -ks & 0 \\ -ks & ks+ka & -ka \\ 0 & -ka & ka \end{bmatrix}\mathbf{x} = \begin{matrix} \mathbf{Fo\ sin\ \omega t} \\ \mathbf{0} \\ \mathbf{0} \end{matrix}$$

Running the state space analysis again will affect in suppression of the amplitude at 1st mode and make it approach to 0. However, it will also result into creation of additional side-bounds due to vibration of absorber mass. To note, the amplitudes of all the peaks should be lower than that of the primary structure's bode plot. Thus, there will be a total of 3 observable peaks or bumps at least signifying 3 DOF. The frequency response plot for a fully-tuned passive system is attached below;

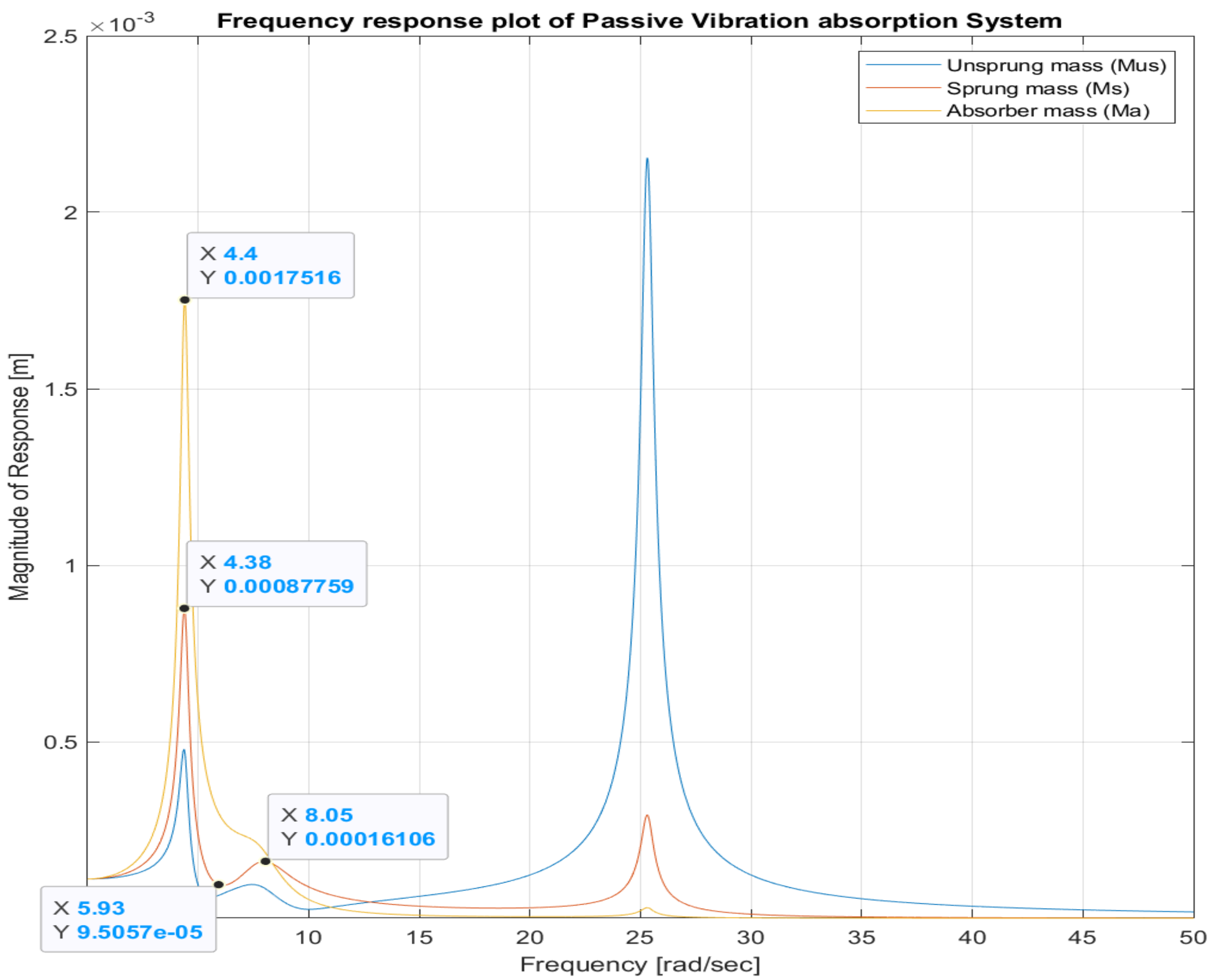

Figure 6.) Frequency response plot of Passive (fully-tuned) system

Here, we observe that the amplitude of the primary system occurring at ω_1 is now approximately equal 0. We also observe that the side-bounds have been created, one at the frequency of 4.41 rad/sec, other one at 8.05 rad/sec. Though the side-bound is not a crisp peak, it's a bump afterall showing one of the modes of vibration. The 3rd mode remains nearly at the same position as in the original case. The highest amplitude of rise is 1.7516 mm, reduction of almost a fifth from the original case's amplitude.

Now let's look at how the amplitudes are posed in a passive vibration absorption system if no damper is attached at absorber mass level.

B.) Passive absorption system without damping;

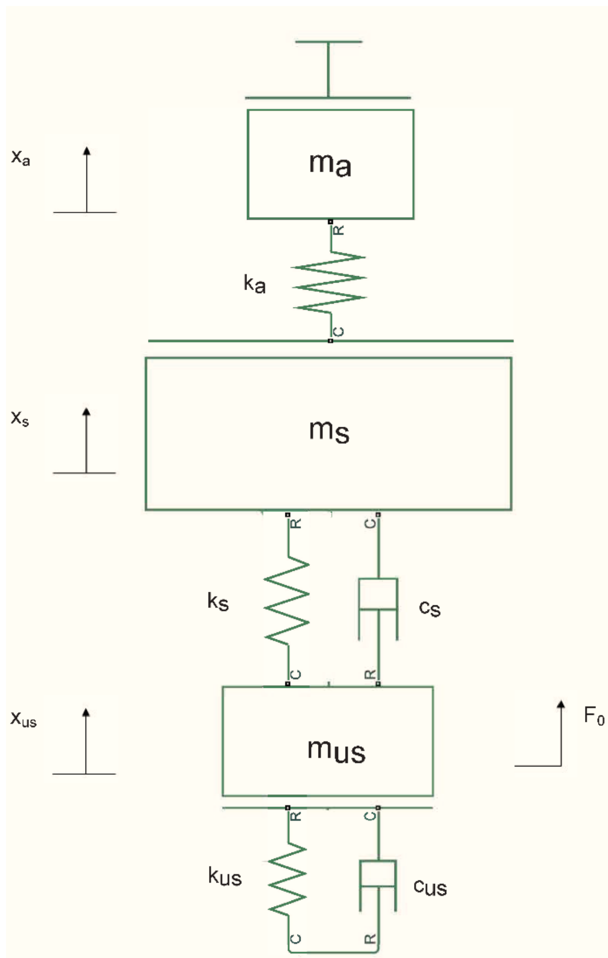

Figure 6.) Schematic diagram - Passive absorber system without damping

One can easily say that is this case, the absence of damper will leave the magnitude of vibration of absorber mass much higher than the Primary structure case. This is because of the fact that the mass of absorber is now only having a certain spring-rate which has been designed from the $\mathbf{m_a}$ & $\boldsymbol{\omega_1}$ values. This absorber will oscillate with all it's stiffness characteristics indefinitely till that particular mode of vibration goes out of phase. Thus, we observe the amplitudes of all the masses to rise rather than fall.

The following equation of motion now changes to the following where last row and column in the damping matrix would basically become 0;

$$\begin{bmatrix} \mathrm{Mus} & 0 & 0 \\ 0 & \mathrm{Ms} & 0 \\ 0 & 0 & \mathrm{Ma} \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} \mathrm{Cus+Cs} & -\mathrm{Cs} & 0 \\ -\mathrm{Cs} & \mathrm{Cs} & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\mathbf{x}} + \begin{bmatrix} kus+ks & -ks & 0 \\ -ks & ks+ka & -ka \\ 0 & -ka & ka \end{bmatrix} \mathbf{x} = \begin{matrix} \mathbf{Fo\ sin\ \omega t} \\ \mathbf{0} \\ \mathbf{0} \end{matrix}$$

This will be clear from the bode plot of the without damping passive system case.

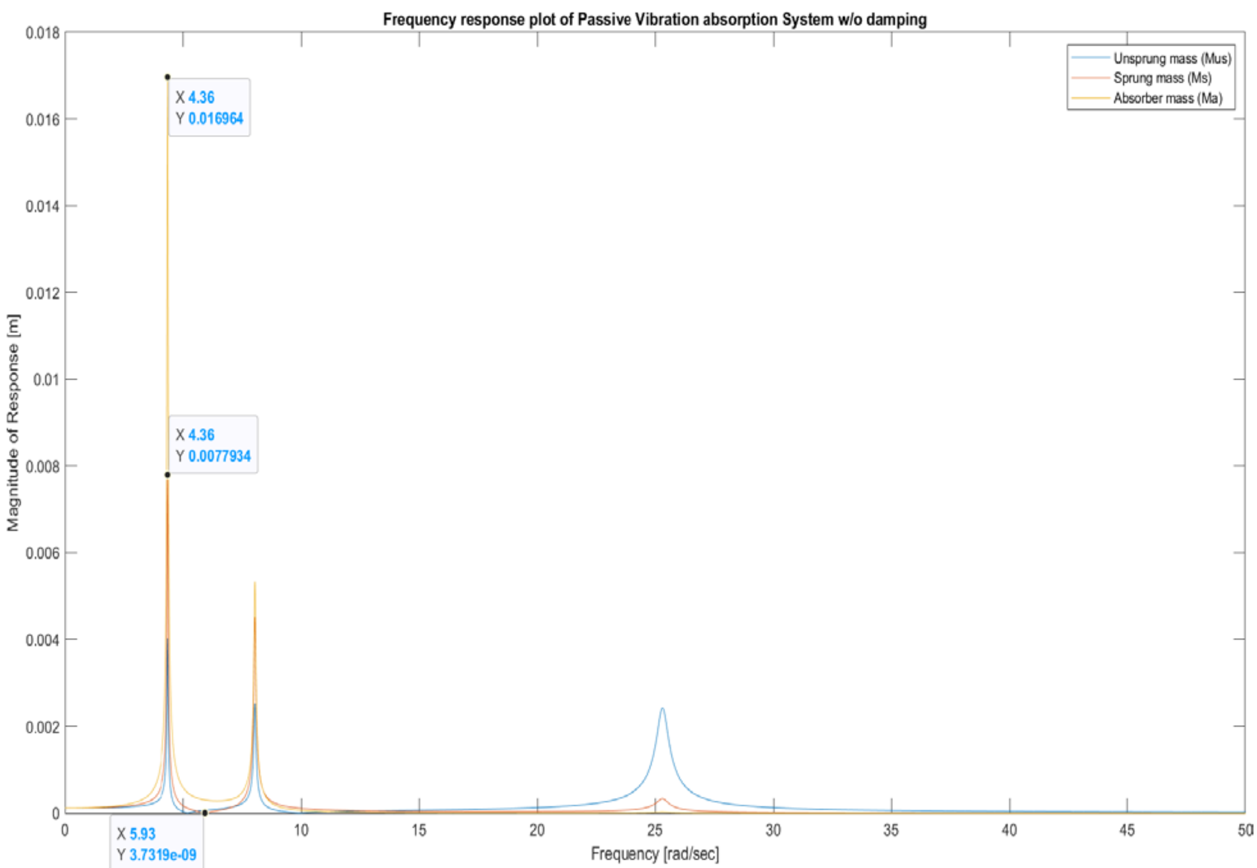

Figure 7.) Frequency response plot of Passive System {w/o damping}

As can be observed, the maximum amplitude undergone by $\mathbf{m_a}$ is 16.964 mm. On comparison, this is a huge rise in the amplitude value from 5.8786 mm of the original case. To achieve lower values of peaking amplitude, the designer will have to choose a spring-rate, 1000 times of the current $\mathbf{k_a}$ rate since all he has to rely on is the spring-rate because the damper is absent. But there are always high costs associated with getting such a stiffness.

Passive and active methods of vibration damping differ in the way they respond to and manage vibrations. A passive system uses simple mechanical devices, fluids, or elastomeric materials, whereas active vibration damping relies on a closed-loop system with measured feedback. Active system aims down at further reducing the overall amplitude of the system with a measured amount of feedback force that opposes the unwanted vibrational motion.

Active Vibration Absorption System

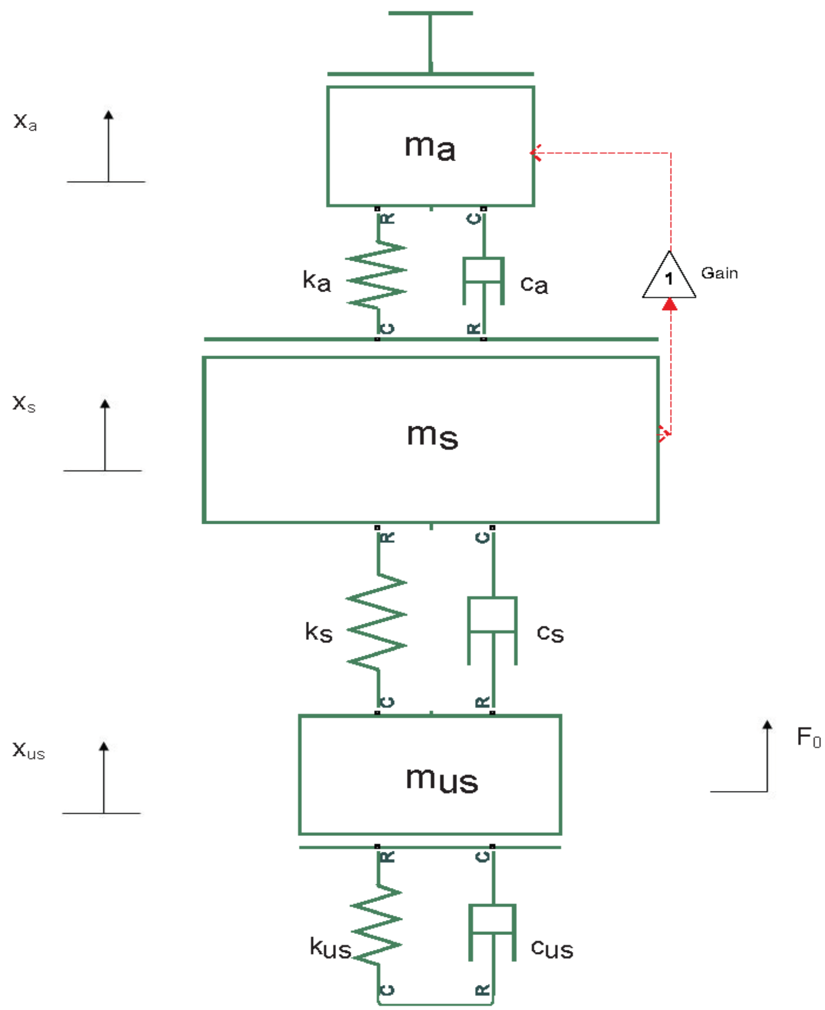

Figure 8.) Schematic diagram – Active vibration absorption system

The following system has been added with a feedback control loop for obtaining a force equal in magnitude and opposite in direction. The above schematic representation of Active control system has a **Gain parameter** which must be tuned in order to receive appropriate feedback force that counters the amplitudes of vibration. The feedback force is given by the relation;

$$fg = -kv * \dot{x}_s$$

where $\boldsymbol{kv}$ is the velocity constant, must be assigned a value proportional to the velocity scale of the vibration of the structure whose vibration needs to be absorbed. We are measuring the velocity constant up against the the scale of vibration since it is feasible to assume the nature of such small scale of vibration as depending on velocity of its motion. We can also tune in terms of displacement constant or acceleration constant provided the nature of activity is in relativistic terms of displacement or acceleration.
A proportional feedback force is going to suppress the amplitude of the primary mode even more. We should keep in mind the stability of the system all the time when tuning with an absorber mass having feedback control loop. One important assumption here is that the state variables are having Continuous linear time-invariant characteristics, i.e.;

$$\dot{x} = \mathbf{A}x(t) + \mathbf{B}u(t)$$
$$y = \mathbf{C}x(t) + \mathbf{D}u(t).$$

We must make a small change in the state matrix [A] whenever there is a feedback system present. That change is given by A_feedback = { A – (B_f*k_v) }.
'A_feedback' is still the state matrix that is the input variable in the state space command in MATLAB. Supplying the necessary state variable in 'ss' command in MATLAB, we solve for the magnitude and phase degree response of the system with respect to frequency. The bode plot for Active case is give below;

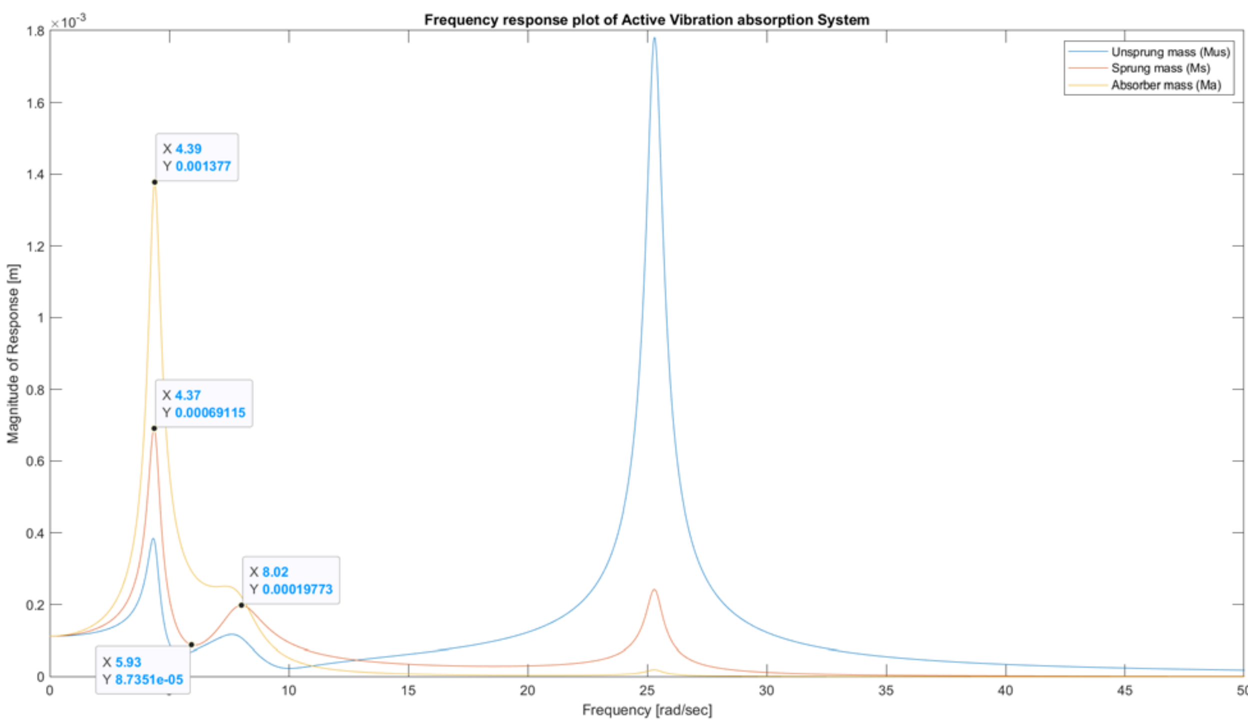

Figure 9.) Frequency response plot of Active vibration absorption system

A noteworthy thing from the above plot is that instead of further decreasing the amplitude of the 1st mode, the amplitude reduces by a very small factor. Also, the system of absorber mass with the feedback loop remains in the stable region of the root locus. Our actual aim of reducing the 1st mode's amplitude and making it to converge to zero has been achieved. The final amplitude of the 1st mode is now 0.08735 mm with the Active system.

The eigenvalues corresponding to the Active absorption system and Passive absorption system are given below.

Passive Vibration Absorption system	-0.2847 +25.3103i -0.2847 -25.3103i -1.2375 + 7.8920i -1.2375 - 7.8920i -0.2067 + 4.3942i -0.2067 - 4.3942i
Active Vibration Absorption system	-0.3445 +25.3094i -0.3445 -25.3094i -0.9834 + 7.9438i -0.9834 - 7.9438i -0.2593 + 4.3826i -0.2593 - 4.3826i

The real part of all the eigenvalues are negative, is what defines stability of the system. We find these values by using **eig(A_p)** & **eig(A_feedback)** function in MATLAB which are our state matrices for the input in $\dot{x}$.

This system can also be evaluated in the Laplace domain instead of time domain. All we require is a transfer function that represents every degree of freedom. For our system with chosen parameters, we have following transfer functions for our primary, passive and active system;

For Primary case:

For mass 1; G1(s)	$0.03333 s^2 + 0.004804 s + 2.5$ --- $s^4 + 0.6245 s^3 + 675.1 s^2 + 93.67 s + 2.25e04$
For mass 1; G2(s)	0.001601 s + 2.5 --- s^4 + 0.6245 s^3 + 675.1 s^2 + 93.67 s + 2.25e04

For Passive case:

For mass 1; G1(s)	0.03333 s^4 + 0.09925 s^3 + 4.17 s^2 + 5.169 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05
For mass 1; G2(s)	0.001601 s^3 + 2.503 s^2 + 5.056 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05
For mass 1; G3(s)	0.003203 s^2 + 5.056 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05

For Active case:

For mass 1; G1(s)	0.03333 s^4 + 0.09925 s^3 + 4.17 s^2 + 5.169 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05
For mass 1; G2(s)	0.001601 s^3 + 2.503 s^2 + 5.056 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05
For mass 1; G3(s)	0.003203 s^2 + 5.056 s + 87.91 --- s^6 + 3.458 s^5 + 726.5 s^4 + 1973 s^3 + 5.522e04 s^2 + 4.829e04 s + 7.912e05

These transfer functions are in s domain, which can be supplied in the ‘ss’ command in MATLAB after making necessary modification to the continuous-time invariant state space models to continuous-time invariant Laplace domain equations.

For the primary system with parameters taken as below;

- unsprung mass, m_{us} = 30 kg;
- sprung mass, m_s = 120 kg;
- Unsprung mass spring-rate, k_{us} = 9000 N/m;
- Sprung mass spring-rate, k_s = 9000 N/m;

we have the following tuned-mass absorber parameters which display optimum vibration suppression characteristics.

Parameters	Passive Absorption System	Active Absorption System
Absorber mass, $\mathbf{m_a}$	50 kg	50 kg
Spring-rate, $\mathbf{k_a}$	1758.6 N/m	1758.6 N/m
Damping coefficient, $\mathbf{c_a}$	[100 - 170] Ns/m	[80 - 150] Ns/m
Maximum vibration Amplitude, **X**	1.7516 mm	1.377 mm
Natural Frequencies, **ω**	ω_1 = 4.399 rad/sec ω_2 = 8.05 rad/sec ω_3 = 25.311 rad/sec	ω_1 = 4.39 rad/sec ω_2 = 8.02 rad/sec ω_3 = 25.3055 rad/sec
Damping factor, **ξ** (for mass m_a) (for mass m_s) (for mass m_{us})	ξ_1 = 0.01124 ξ_2 = 0.1549 ξ_3 = 0.0469	ξ_1 = 0.0143 ξ_2 = 0.2165 ξ_3 = 0.0820

We can assertively say that with increasing the damping factor, there is going to be much higher damping and the system will have a highly suppressed amplitude. From the graph of Transmissibility ratio vs damping values **ξ** given below, we find that as we increase the **ξ** value, the asymptote gradually gets near to 1 as compared to very low values like 0.01 where the frequency Transmissibility ratio **TR** is converging to infinity. A TR -> ∞, we have resonance conditions.

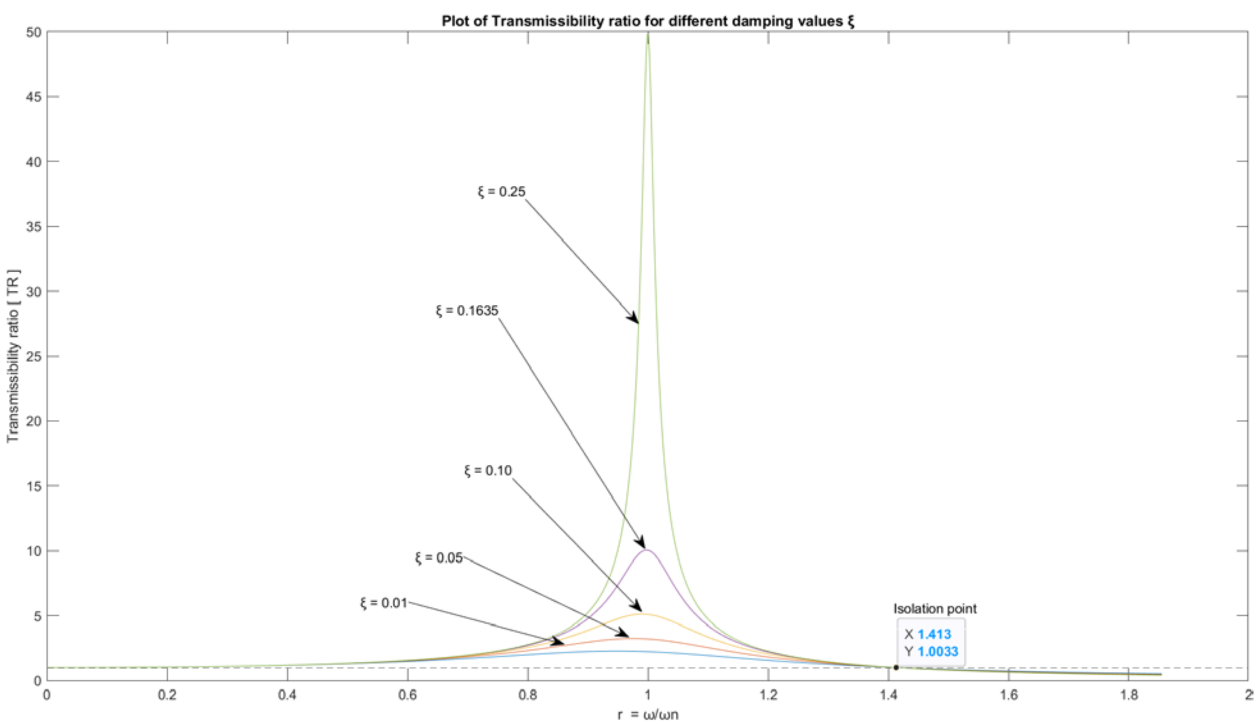

Figure 9.) Transmissibility Ratio for different damping values ξ

Observations and conclusions from the tuning process

- With the above listed variables, the whole vibration suppression scheme is working the best. Owing to the limitation of absorber mass's value, when we try to achieve the objective of suppressing the value of 1st mode of vibration of our original system. In doing that, we would observe that as we shift the value of damping coefficient, c_a to the left of the above stated value, the amplitude peaks for the Passive case will go higher than the primary structure's maximum amplitude. Also, instead of creating two side-bounds, one of the modes will get overlapped on the other completely, and hence the plots look unacceptable. The above value of c_a is the bandwidth where one can play around. But it should always be the case that the c_a of the Passive case should be set higher that c_a Active case because we do not require a high value of c_a in active case since there is feedback system present which will contribute much higher in generating the opposing force.

- Changing the gain value K_v in the range of 10 to 30 works well in lowering the amplitudes and also within this range, the real part of the eigenvalues is negative, i.e., in the stable region. Below and above this value, the eigenvalue's real part runs into the positive real axis region thus making the system unstable.

- Values of masses are not very symmetric; one is 30 kgs, other is 120 kgs. Also, for an automobile, the practical values of stiffnesses are of the order of 10^5, 10^6. For the sake of analysis, here we are not adopting such high values of stiffness as it would not have any visible result improvement because they are the industry standard values. Same goes with the value of mass. Assuming it in the limit of 1/10th or 1/7th of primary mass is not going to show any good results especially because of the fact that masses values are highly unsymmetrical.

Here are the final set of parameters chosen for the purpose of this analysis;

Unsprung mass, m_{us}	30 kg
Sprung mass, m_s	120 kg
Spring-rate, k_{us}	9000 N/m
Spring-rate, k_s	9000 N/m
Damping coefficient (for Passive), c_a	100 Ns/m
Damping coefficient (for Active), c_a	90 Ns/m
Damping factor for primary structure, ξ	1% (case of light damping in the system)

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APPENDIX

Nomenclature

m_s Mass of sprung weight
m_{us} Mass of unsprung weight
m_a Mass of absorber
k_s Spring-rate of sprung weight
k_{us} Spring-rate of unsprung weight
k_a Spring-rate of absorber
c_s Damping coefficient at sprung weight
c_{us} Damping coefficient at unsprung weight
c_a Damping coefficient at absorber mass
ξ Damping factor in the system
Kv Velocity constant
f_g Feedback control force
TR Transmissibility ratio
r frequency ratio
ω frequency, rad/sec

MATLAB code to perform State Space Analysis of design for vibration suppression

```
clc;clear all

%% %%%%%%    PART B   %%%%%%%%%%%
% 1.)))))) Primary structure with a unit force input at m1.
% Setting up the system values for mass and springs at different levels.

% m1 = unsprung mass, m2 = sprung mass, k1 & k2 = stiffness accordingly
m1 = 30; m2 = 120; k1 = 9000; k2 = 9000; ma = 50;
m = [m1 0;0 m2]; k = [k1+k2 -k2;-k2 k2];

% This section computes the values of constants 'alpha' & 'beta' to
be ...% used for proportional damping.
            Minv = [1/sqrt(m1) 0;0 1/sqrt(m2)];
            KTilda = Minv*k*Minv
            [EigVec,EigValues] = eig(KTilda)
            v1 = sqrt(EigValues(1,1))
            v2 = sqrt(EigValues(2,2))

            r = [0.02;0.02]; a = [1/v1 v1;1/v2 v2];
            constants = a\r;
            alpha = constants(1)
            beta = constants(2)
% Now describing the state space variables;
c = alpha*m + beta*k  %%%% Proportional damping for the structure only.

Bf= [1;0]; %%% Assuming that the force is acting at unsprung mass level.
A = [zeros(2) eye(2); -inv(m)*k -inv(m)*c];
B = [zeros(2,1);inv(m)*Bf];
C = [eye(2) zeros(2)];
D = zeros(2,1);
eig(A)
w = 0:0.01:50; % Giving the range of observation of natural frequencies.

sys=ss(A,B,C,0)
[mag,phase] = bode(sys,w);
figure(1)
plot(w,mag(1,:),w,mag(2,:));
legend('Unsprung Mass','Sprung Mass')
xlabel('Frequency [rad/sec]'); ylabel('Magnitude of Response [m]')
title('Frequency response plot of Primary Structure');
legend('Unsprung mass (Mus)','Sprung mass (Ms)');

% %%%%% In laplace domain

P1 = bodeoptions;
P1.MagScale = 'linear';
P1.MagUnits = 'db';

grid on;
hold on
[n1,d1] = ss2tf(A,B,C,D)
G1 = tf(n1(1,:),d1)
G2 = tf(n1(2,:),d1)

%  bodeplot(G1,P1)
% bodeplot(G2,P1)
```

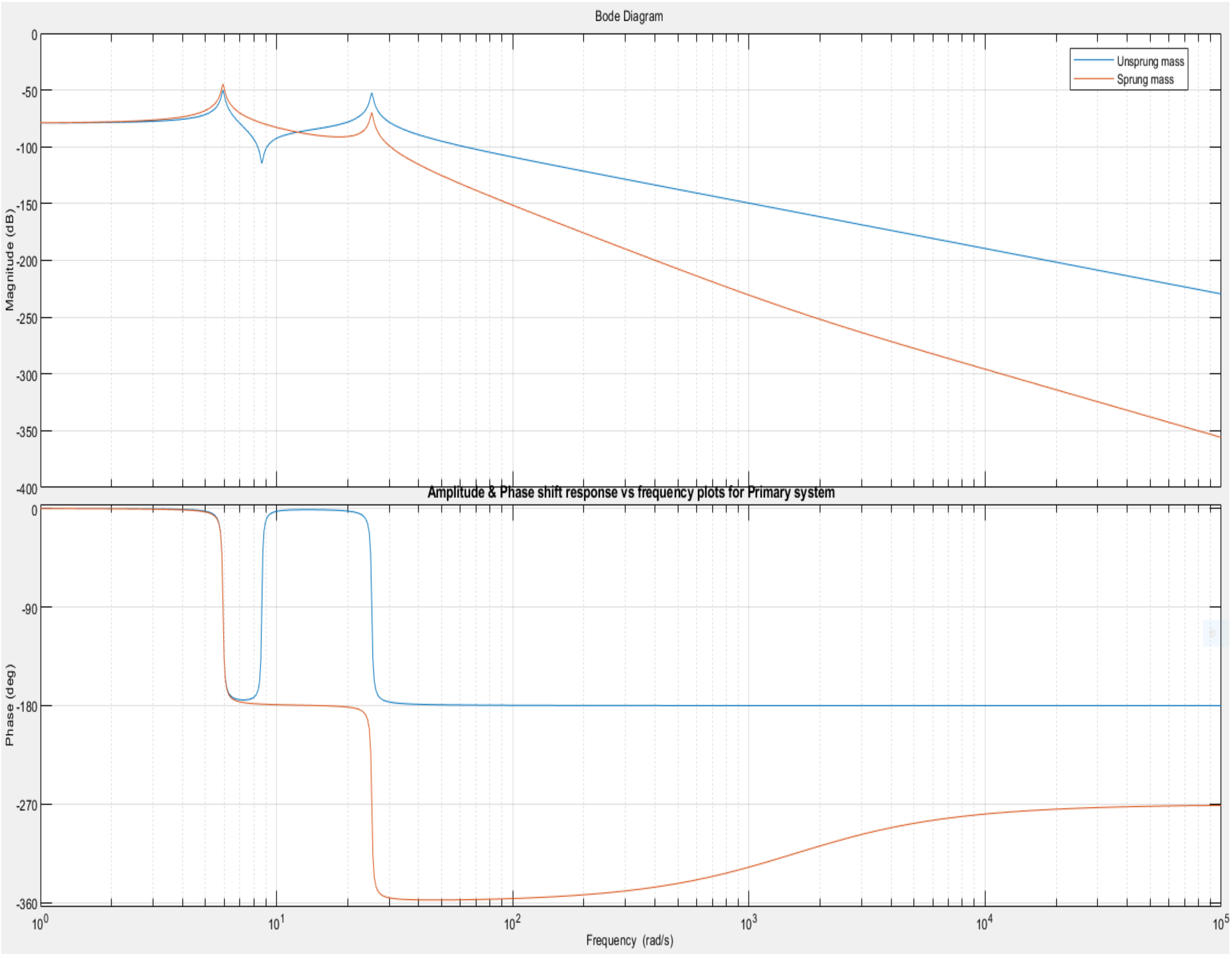
```
%% 2.)))))))  System with added absorber mass 'ma' - [Passive system]

% We set the value of spring-rate at the absorber mass as given below.% %
min(v1,v2) gives me the 1st mode that i want to suppress
ka = ((min(v1,v2))^2)*(ma)   %%% Since we are suppressing the first mode.
M = [m1 0 0;0 m2 0;0 0 ma]; K = [k1+k2 -k2 0;-k2 k2+ka -ka;0 -ka ka];

% Assuming damping ratio for passive case to be around 16.35 %
ca = 100;  %% Damping coefficient for the Passive Absorption case.
% % Damping matrix for overall Passive Absorption system case.
c_p = [c(1,1) c(1,2) 0;c(2,1) c(2,2)+ca -ca;0 -ca ca]

Bf_p= [1;0;0]; D_p = zeros(6,1);

A_p = [zeros(3) eye(3); -inv(M)*K -inv(M)*c_p];
B_p = [zeros(3,1);inv(M)*Bf_p];
C_p = [eye(3) zeros(3)];
SYS=ss(A_p,B_p,C_p,0)
[Mag,Phase] = bode(SYS,w);
figure(2)
plot(w,Mag(1,:),w,Mag(2,:),w,Mag(3,:))
xlabel('Frequency [rad/sec]'); ylabel('Magnitude of Response [m]');
title('Frequency response plot of Passive Vibration absorption System');
legend('Unsprung mass (Mus)','Sprung mass (Ms)','Absorber mass (Ma)');

% %%% In Laplace domain;

P2 = bodeoptions;
P2.MagScale = 'linear';
P2.MagUnits = 'db';

grid on;
hold on
[n2,d2] = ss2tf(A_p,B_p,C_p,zeros(3,1))
G_1 = tf(n2(1,:),d2)
G_2 = tf(n2(2,:),d2)
G_3 = tf(n2(3,:),d2)
% bodeplot(G_1,P2) %
bodeplot(G_2,P2)

%% 3.)))))  System with added feedback gain loop - [Active System]

KV = 30; %%%% Optimal value of feedback gain 'KV' ranges from  9<kv<30
kv = -[0 0 0 0 KV 0];% State Feedback gain vector
Bf_p= [1;0;0];
D = zeros(6,1);
Bf_gain= [1;0;-1];

ca_a = 90; % Damping coefficient at absorber mass level for Active
System% Damping matrix for the overall system in Active absorption case
C_P = [c(1,1) c(1,2) 0;c(2,1) c(2,2)+ca_a -ca_a;0 -ca_a ca_a];
% % State Variables for the Active case that has feedback
A_a = [zeros(3) eye(3); -inv(M)*K -inv(M)*C_P];
B_p = [zeros(3,1);inv(M)*Bf_p];
B_gain = [zeros(3,1);inv(M)*Bf_gain];
```

```
A_feedback = (A_a - (B_gain*kv));
C = [eye(3) zeros(3)];

SYS_a=ss(A_feedback,B_p,C,0)
[mag_a,phase_a] = bode(SYS_a,w);
figure(3)
plot(w,mag_a(1,:),w,mag_a(2,:),w,mag_a(3,:));
title('Frequency response plot of Active Vibration absorption System');
xlabel('Frequency [rad/sec]'); ylabel('Magnitude of Response [m]');
legend('Unsprung mass (Mus)','Sprung mass (Ms)','Absorber mass (Ma)');
%
% % %%%% In Laplace Domain;

P3 = bodeoptions;
P3.MagScale = 'linear';
P3.MagUnits = 'db';%% Unit of Amplitude axis.'dB' - decibels ,'abs' - meter

grid on;
hold on
[n3,d3] = ss2tf(A_p,B_p,C_p,zeros(3,1))
g1 = tf(n3(1,:),d3)
g2 = tf(n3(2,:),d3)
g3 = tf(n3(3,:),d3)
% % bodeplot(g1,P3)
% % bodeplot(g2,P3)
% % bodeplot(g3,P3)
%
eig(A_p)
eig(A_feedback)

%% Plotting of Transmissibility Ratio for different damping factor values
wn = 5.93; %% frequency of 1st mode of vibration
w = linspace(0,11,1000);
r = w/wn;
% damping proportion values

z1 = 0.25;
z2 = 0.1635;
z3 = 0.10;
z4 = 0.05;
z5 = 0.01;

    F1 = sqrt(((1+(2*z1*r).^2))./(((1-r.^2).^2 + (2*z1*r).^2)));
F2  =  sqrt(((1+(2*z2*r).^2))./(((1-r.^2).^2  +  (2*z2*r).^2)));
F3  =  sqrt(((1+(2*z3*r).^2))./(((1-r.^2).^2  +  (2*z3*r).^2)));
F4  =  sqrt(((1+(2*z4*r).^2))./(((1-r.^2).^2  +  (2*z4*r).^2)));
F5 = sqrt(((1+(2*z5*r).^2))./(((1-r.^2).^2 + (2*z5*r).^2)));
figure(4)
plot(r,F1,r,F2,r,F3,r,F4,r,F5);yline(1,'--');
title('Plot of Transmissibility ratio for different damping values ξ')
```

Amplitude and Phase shift vs frequency plots

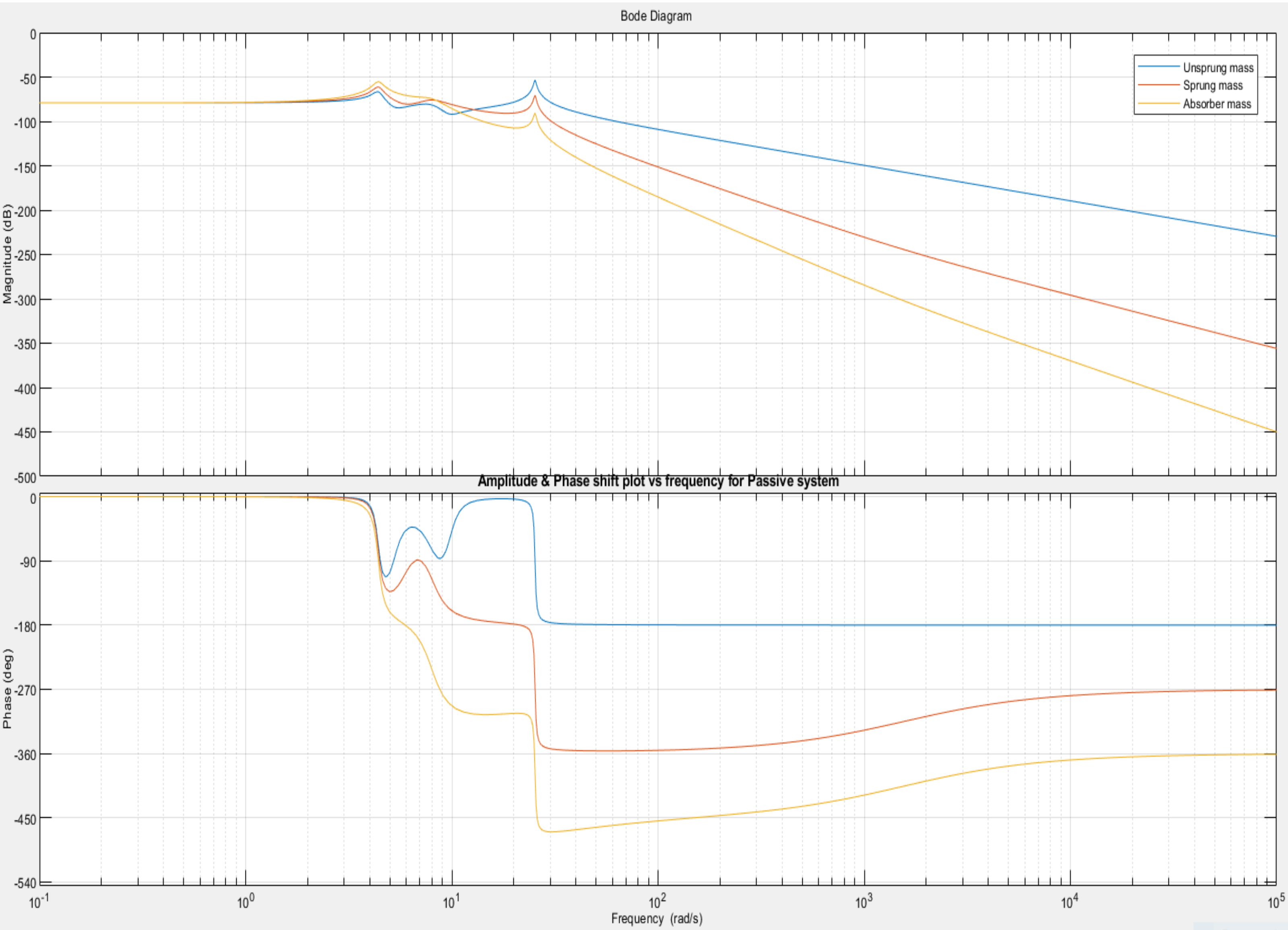
This plot is on a log-log scale where the magnitude of vibration is expressed in terms of 'dB' decibels. The phase shift diagram is an important piece of information as well. Phase shift is seen whenever the structures hits its one of the many modes of vibrations. It is evident from the graphs below.

Primary system

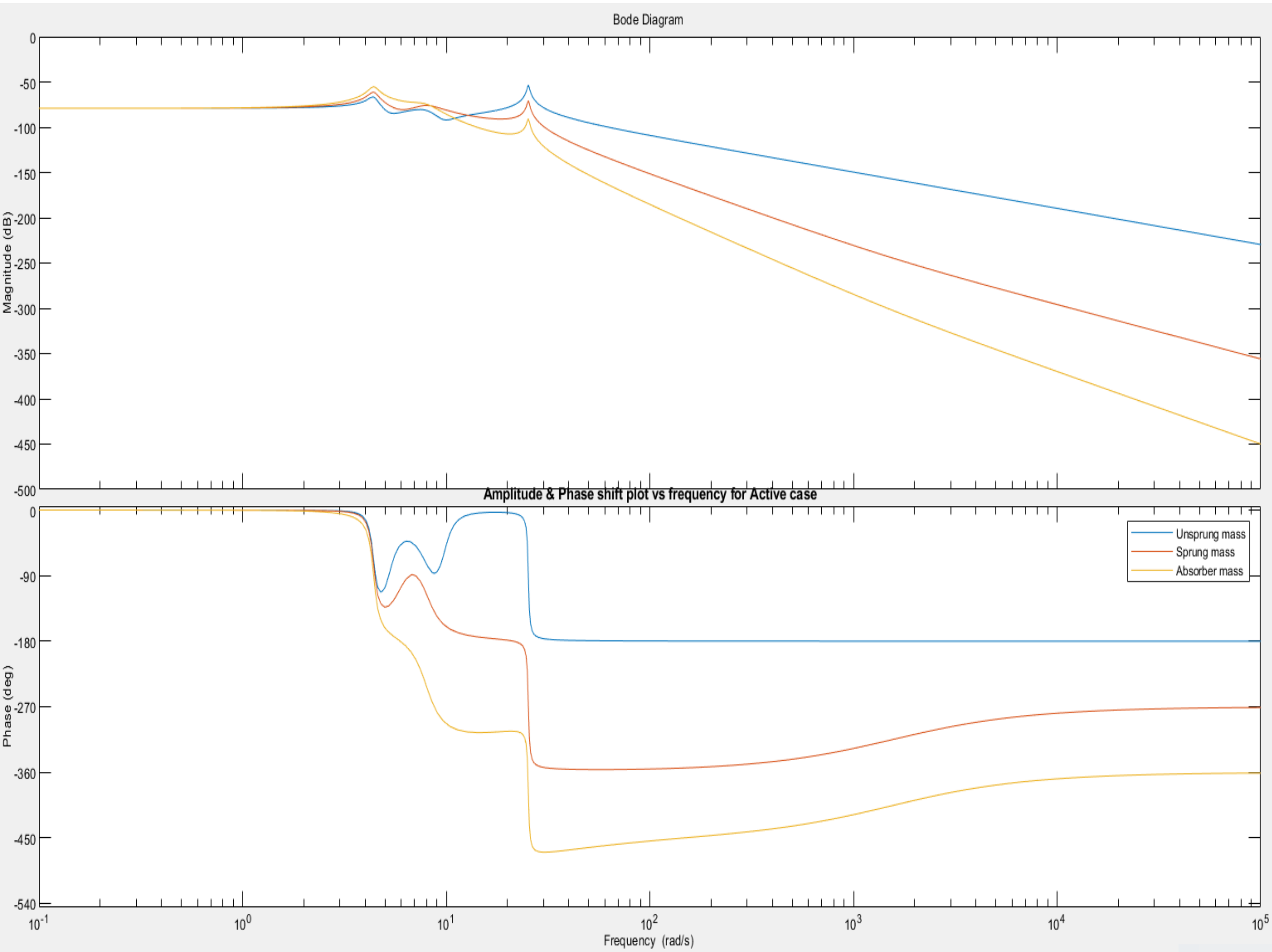


Also, it is noteworthy from the phase shift graph that until the 1st mode of the structure's vibration, the masses will be in phase sync with each other. After passing the 1st mode of vibration the masses follow different phase degrees.

Passive case



Active case


Bode Diagram
Magnitude (dB)
0
-50
-100
-150
-200
-250
-300
-350
-400
-450
-500
Amplitude & Phase shift plot vs frequency for Active case
Phase (deg)
0
-90
-180
-270
-360
-450
-540
Unsprung mass
Sprung mass
Absorber mass
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
Frequency (rad/s)